MODELING OF UNSTEADY WAVE PROCESSES IN MOVING MEDIA

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Using the nonreciprocity relation a difference scheme is built for the wave equation in a moving medium. Examples are given of solution of the problems of sound propagation in the presence of wind.

In problems of acoustics, electrodynamics, and heat conduction involving intense heat release momentum transfer is accompanied, as a rule, by medium motion, and, therefore, the corresponding hyperbolic equations must be solved in a moving medium.

The processes under consideration are described by a wave equation of the form

$$\rho(P, x, t)\frac{\partial^2 P}{\partial t^2} = \frac{\partial}{\partial x}\left(k(P, x, t)\frac{\partial P}{\partial x}\right), \tag{1}$$

where medium motion must be taken into account. The modified linear wave equation, when $\rho = 1/C^2$, $k = 1$, in a moving medium has been considered, with some limitations, by D. I. Blokhintsev [1, 2]. In [3-5], equations are derived that have a wider field of application but, unfortunately, are rather complicated for calculations.

The present work is devoted to modeling of sound propagation in a moving inhomogeneous medium by constructing a difference scheme that accounts for the medium motion.

It is known [2] that downstream and upstream disturbances are propagated at different velocities. This effect will be accounted for when constructing a difference scheme for wave Eq. (1) with the aid of deformation of a difference cell.

We shall consider the interrelationship of space and time measurements in a moving medium. Let us introduce a uniform network in the medium at rest $\omega_{h_0,\tau} = \omega_{h_0}\times\omega_\tau$, where $\omega_{h_0} = \{x_i = ih_0,\ i = 0, 1, 2, \ldots, N_x,\ h_0N_x = l\}$, $\omega_\tau = \{t_j = j\tau,\ j = 0, 1, 2, \ldots, N_\tau,\ \tau N_\tau = T\}$ (Fig. 1a). In the case of medium motion, each node replaces a relatively motionless observer with velocity V_i, and, therefore, for the time of signal propagation, within which the observer is continuously recording the amplitude and frequency of pressure or flow-velocity variation, the node is displaced by same additional distance (Fig. 1b). For the motionless observer at node i sending or receiving a signal (a change in pressure or velocity), downstream and upstream paths of signal transmission for a cell with length h_0 (h_0 is the cell length in the Lagrange reference system) will be different.

We now consider two cases: a) signal propagation to the right of node i and b) signal propagation to the left of node i. Assume that the flow velocity within the limits of the difference cell as well as the wave velocity are constant quantities. The dependence of the medium velocity V and the sound velocity C on the coordinate and time will be taken into account in the difference scheme.

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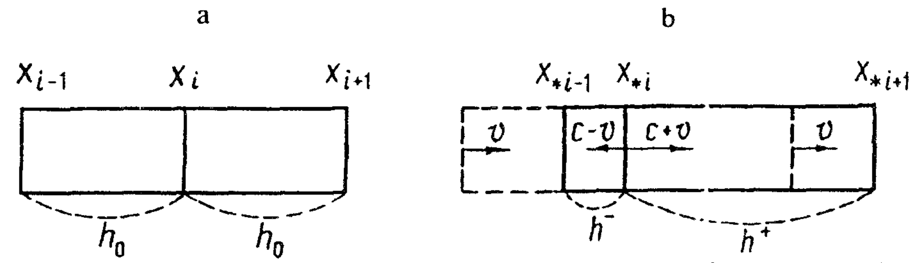

Fig. 1. Deformation of a difference cell: a) medium at rest; b) moving medium.

1. Let a motionless observer be at the node i. Then a signal sent downstream has total velocity $C + V$, and for the time of signal propagation Δt from node i to node $i + 1$, node $i + 1$ itself is displaced by an additional distance $V\Delta t$; therefore (Fig. 16)

$$h^+ = h_0 + V\Delta t,$$

where

$$\Delta t = h^+/(C + V), \quad h^+ = h_0 + Vh^+/(C + V).$$

2. A motionless observer is at the node i. Then a signal sent upstream has total velocity $C - V$, and for the time of signal propagation Δt from node i to node $i - 1$, the latter approaches the former by distance Δt; therefore (Fig. 1b)

$$h^- = h_0 - V\Delta t,$$

where

$$\Delta t = h^-/(C - V), \quad h^- = h_0 - Vh^-/(C - V).$$

Whence

$$h^+ = h_0\left(1 + \frac{V}{C}\right), \quad h^- = h_0\left(1 - \frac{V}{C}\right), \tag{2}$$

where h^+ and h^{-1} are the cell dimensions in the Euler reference system. Consequently, for a motionless observer at node i in the Euler reference system the initially uniform network undergoes deformation.

For the difference scheme, we apply relations (2), which lead, in fact, to deformations of the Euler calculation region due to medium motion.

The size of the cell (body) must be independent of the sign of the velocity; therefore, in constructing a difference scheme we shall use the condition of independence of cell size of the direction and the velocity sign of the moving medium. This means that a signal sent by an observer at time $j - 1$ downstream (or upstream) must be received at moment $j + 1$ at the same node i when the signal direction is reversed, i.e., the measurements become time-averaged. In other words, the procedure of time averaging of measurements reflects the fact that one and the same node is an emitter and a receiver of waves at the same moment in time.

The constructing the difference scheme, we assume that wave Eq. (1) holds for a medium at rest and for a moving medium in a local comoving Lagrange reference system for a given individual difference cell with a sufficiently small step h_0. Having solved (2) for h_0, we pass to Euler variables and select a space step equal, for convenience, to $h = l/N_x$ and obtain

$$h_0 = h/(1 \pm V/C),$$

$$\bar{h}_0 = \frac{1}{2}\left(\frac{h}{\alpha} + \frac{h}{\gamma}\right) = h/(1 - V^2/C^2), \tag{3}$$

where $\alpha = 1 + V/C$, $\gamma = 1 - V/C$.

We write the initial wave equation not for a segment but for difference spatial nodes of the network

$$\rho(P, x, t) \frac{\partial^2 P}{\partial t^2} = \frac{\partial}{\partial x} \left(k(P, x, t) \frac{\partial P}{\partial x} \right) \Bigg|_{x=x_*}, \quad t_{j-1} \le t \le t_{j+1}, \tag{4}$$

where x_* are the deformed nodes built in accordance with relations (3).

The boundary and initial conditions are as follows

$$P(0, t) = P_1(t), \quad P(l, t) = P_2(t),$$

$$P(x, t_0) = P_0(x), \quad \frac{\partial P(x, t_0)}{\partial t} = P^*(x), \quad 0 \le x \le l.$$

We designate an approximate value of the function P at the nodes $\omega_{h_C,\tau}$ by $y_{i,j}$. Equation (4) will be approximated by the scheme with allowance for nonreciprocity relation (2) and independence of the mesh width from the direction of medium motion (time-averaging of measurements):

$$\rho(y_{i,j}, x_i, t_j) \frac{y_{i,j+1} - 2y_{i,j} + y_{i,j-1}}{\tau^2} = \frac{(\alpha\gamma)_{i,j}}{2h} \left(a_{i+1,j+1} \frac{y_{i+1,j+1} - y_{i,j+1}}{h} - \right.$$

$$- b_{i,j+1} \frac{y_{i,j+1} - y_{i-1,j+1}}{h} + b_{i+1,j-1} \frac{y_{i+1,j-1} - y_{i,j-1}}{h} -$$

$$\left. - a_{i,j-1} \frac{y_{i,j-1} - y_{i-1,j-1}}{h} \right), \quad i = 2, \overline{N_x - 1}, \quad j = 1, \overline{N_t - 1}, \tag{5}$$

where

$$a_{i,j} = \frac{\alpha_{i,j} k(y_{i,j}, x_i, t_j) + \alpha_{i-1,j} k(y_{i-1,j}, x_{i-1}, t_j)}{2};$$

$$b_{i,j} = \frac{\gamma_{i,j} k(y_{i,j}, x_i, t_j) + \gamma_{i-1,j} k(y_{i,j}, x_i, t_j)}{2}.$$

The conditions at the boundaries are

$$y_{1,j+1} = P_1(t_j), \quad y_{N_x,j+1} = P_2(t_j), \quad j = 1, \overline{N_t - 1}. \tag{6}$$

The initial conditions are

$$y_{i,0} = P_0(x_i), \quad i = 0, \overline{N}_x,$$

$$y_{i,1} = P_0(x_i) + \tau P^*(x_i), \quad i = 1, \overline{N_x - 1}. \tag{7}$$

Difference scheme (5) is implemented in the following iteration process

$$\rho(\overset{s}{y}_{i,j}, x_i, t_j) \frac{\overset{s+1}{y}_{i,j+1} - 2y_{i,j} + y_{i,j-1}}{\tau^2} = \frac{(\alpha\gamma)_{i,j}}{2h} \left(\overset{s}{a}_{i+1,j+1} \frac{\overset{s+1}{y}_{i+1,j+1} - \overset{s+1}{y}_{i,j+1}}{h} - \right.$$

$$\left. - \overset{s}{b}_{i,j+1} \frac{\overset{s+1}{y}_{i,j+1} - \overset{s+1}{y}_{i-1,j+1}}{h} + b_{i+1,j-1} \frac{y_{i+1,j-1} - y_{i,j-1}}{h} - a_{i,j-1} \frac{y_{i,j-1} - y_{i-1,j-1}}{h} \right), \tag{8}$$

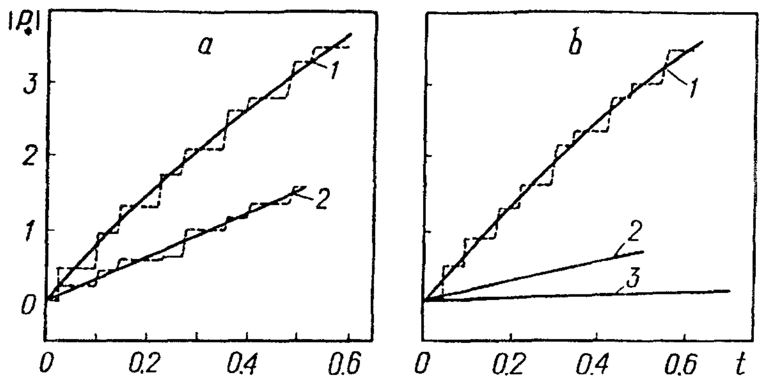

Fig. 2. Amplitude of the acoustic pressure vs time: $\omega = 800\ \text{sec}^{-1}$, a) $x = 0.33$: 1) $V = 0$; 2) 10 m/sec; b) $x = 0.66$: 1) $V = 0$; 2) 10 m/sec; 3) 20 (dashed line, the calculated amplitude value; solid line, the time-averaged absolute amplitude value). $|P_*|$, Pa; t, sec.

$$\overset{s+1}{y}_{1,j+1} = P_1(t_{j+1}), \quad \overset{s+1}{y}_{N_x,j+1} = P_2(t_{j+1}), \quad s = 0, 1, 2, \ldots,$$

where, for instance:

$$\overset{s}{a}_{i,j} = \frac{\alpha_{i,j}k(\overset{s}{y}_{i,j}, x_i, t_j) + \alpha_{i-1,j}k(\overset{s}{y}_{i-1,j}, x_{i-1}, t_j)}{2},$$

$$\overset{s}{b}_{i,j} = \frac{\gamma_{i,j}k(\overset{s}{y}_{i,j}, x_i, t_j) + \gamma_{i-1,j}k(\overset{s}{y}_{i,j}, x_i, t_j)}{2}.$$

Iteration is carried out until the following condition is fulfilled

$$|\overset{s+1}{y}_{i,j+1} - \overset{s}{y}_{i,j+1}| < \varepsilon_1 |\overset{s}{y}_{i,j+1}| + \varepsilon_2, \quad s = 0, 1, 2, \ldots$$

The values of each successive approximation are found by the elimination technique. It is easy to verify that the elimination algorithm is stable for any h and τ.

Examples of Numerical Solution of the Wave Equation in Moving Media. A flat infinite membrane vibrating in a gas causes periodic gas compression and rarefaction around it and thus is a source of acoustic waves. Consider the case when a membrane performs harmonic oscillations of the type $A \sin(\omega t)$ and the pressure at distance l is equal to zero. For instance, such a problem can be observed when acoustic waves propagate along a narrow long tube of constant cross-section whose width is small as compared with wavelength. Under these conditions we can assume that all quantities (velocity, density, pressure, etc.) are constant in each tube cross section and the direction of wave propagation coincides with that of the tube axis. The pressure difference between the gas at the tube end and the surrounding space is small as compared with the pressure difference inside the tube. Therefore with a sufficient degree of accuracy, we assume equality of the gas pressure to zero as a boundary condition [7].

In the presence of wind $V(x, t)$ along the axis x, the pressure satisfies wave Eq. (4) with boundary conditions

$$P(0, t) = A \sin(\omega t), \quad P(1, t) = 0,$$

and initial conditions

$$P(x, 0) = 0, \quad \left.\frac{\partial P(x, t)}{\partial t}\right|_{t=0} = 0, \quad 0 \le x \le 1.$$

The initial conditions correspond to a medium at the rest at the initial moment of time. For the example under consideration, Eq. (1) acquires a linear form with the coefficients $\rho = 1/C^2$, $k = 1$. In this case, a solution is sought

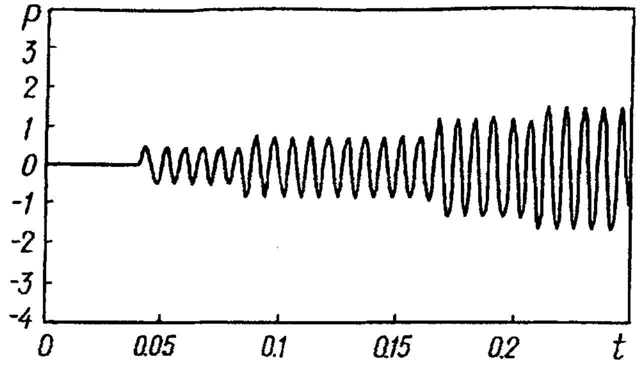

Fig. 3. Acoustic pressure vs time at $V = 0$, $x = 0.66$, $\omega = 800\ \text{sec}^{-1}$. P, Pa.

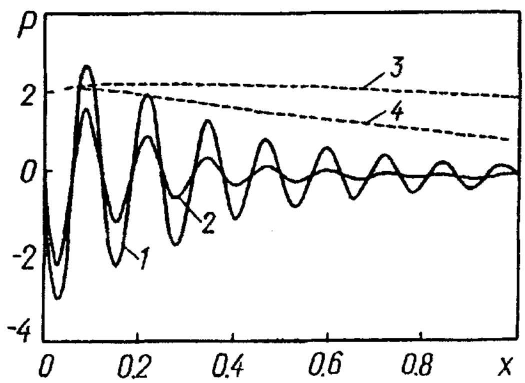

Fig. 4. Acoustic pressure vs the coordinate: 1) $V = 10$ m/sec; 2) 20 m/sec at $t = 0.4002$ sec, $\omega = 800\ \text{sec}^{-1}$; 3) $V = 0$, and 4) $V = -3x + 5$ at $t = 0.\ 3$ sec, $\omega = 900\ \text{sec}^{-1}$; curves 3, 4) the absolute amplitude value.

directly from approximation relations of type (8) at $s = 0$. Calculations were carried out for different space and time steps.

Figure 2 shows the time-dependent amplitudes of the acoustic pressure at different gas velocities. As seen, with an increase in gas velocity, the build-up of pressure amplitude decreases. In this case, the most pronounced decrease in the amplitude and, consequently, in the intensity of acoustic radiation is observed at points far from the membrane (Fig. 2b). Figure 3 shows pressure versus time at the point $x = 0.66$ at $V = 0$. In the pressure of velocity, the dependence preserves its character. In the calculations, we adopted the following values of the parameters: $A = 0.5$ Pa, $l = 20$ m, $C = 320$ m/sec, where C is the sound velocity.

The dependence of acoustic pressure on the coordinate is shown in Fig. 4, which is also indicative of a decrease in the amplitude and, consequently, in the intensity of radiation of acoustic waves with an increase of gas velocity, since the medium under consideration does not dissipate and energy dissipation proceeds only due to gas motion. Curves 3 and 4 are obtained for the case when the gas velocity depends linearily on the coordinate and is directed along the axis x. According to [2], sound audibility decreases in the presence of wind. Naturally, in a real atmosphere this effect can be associated with atmosphere turbulence, but it has been also observed in the presence of weak wind (1–2 m/sec).

We consider one more problem, i.e., the occurrence of sound vibrations in a resonator. The simplest examples of such resonators are tubes open on one or both ends, Helmholtz resonators (in the form of bottles), etc. It is easy to make all resonators of such kind "sound" in an air flow by blowing over their mouth. An open end is identical to an absolutely soft cover when the pressure (acoustic not atmospheric) is practically equal to zero. At the left-hand boundary of the resonator $x = 0$ we have the condition $P(0, T) = 0$; at the right-hand boundary

$$\left.\frac{\partial P(x, t)}{\partial x}\right|_{x=1} = 0 .$$

Let the tube have intrinsic noise with a set of harmonics. Then in the presence of wind whose velocity is directed along the resonator axis the phenomenon of resonance accompanied by a strong build-up of wave amplitude and sound enhancement has been observed.

The results of calculations for sound attenuation in the presence of wind and sound enhancement in a resonator tube, with air flowing in it, correspond to experimental data [2].

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